Multi-step-ahead prediction of volatility proxies

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Though machine learning techniques have been often used for stock prices forecasting, few results are available for market fluctuation prediction. Nevertheless, volatility forecasting is an essential tool for any trader wishing to assess the risk of a financial investment. The main challenge of volatility forecasting is that, since this quantity is not directly observable, we cannot predict its actual value but we have to rely on some observers, known as volatility proxies (Poon & Granger, 2003) based either on intraday (Martens, 2002) or daily data. Once a proxy is chosen, the standard approach to volatility forecasting is the well-known GARCH-like model(Andersen & Bollerslev, 1998). In recent years several hybrid approaches are emerging (Kristjanpoller et al., 2014; Dash & Dash, 2016; Monfared & Enke, 2014) which combine GARCH with a non-linear computational approach. What is common to the state-of-the art is that volatility forecasting is addressed as an univariate and one-step-ahead autoregressive (AR) time series problem.

The purpose of our work is twofold. First, we aim to perform a statistical assessment of the relationships among the most used proxies in the volatility literature. Second, we explore a NARX (Nonlinear Autoregressive with eXogenous input) approach to estimate multiple steps of the output given the past output and input measurements, where the output and the input are two different proxies. In particular, our preliminary results show that the statistical dependencies between proxies can be used to improve the forecasting accuracy.

1. Background

Three main types of proxies are available in the literature : the proxy $\sigma^{SD,n}$, the family of proxies σ^i and σ^G. The first proxy corresponds to the natural definition of volatility (Poon & Granger, 2003), as a rolling standard deviation over a past time window of size n

$$\sigma_t^{SD,n} = \sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}(r_{t-i} - \bar{r_n})^2}$$

where $r_t = \ln\left(\frac{P_t^{(c)}}{P_{t-1}^{(c)}}\right)$ is the daily continuously compounded return , $\bar{r_n}$ is the average over $\{t, \cdots, t-n+1\}$ and $P_t^{(c)}$ are the closing prices. The family of proxies σ_t^i is analytically derived in Garman and Klass (1980).

The proxy $\sigma_t^G = \sqrt{\omega + \sum_{j=1}^{p}\beta_j(\sigma_{t-j}^G)^2 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2}$ is the volatility estimation returned by a GARCH (1,1) (Hansen & Lunde, 2005) where $\varepsilon_{t-i} \sim \mathcal{N}(0,1)$ and the coefficients $\omega, \alpha_i, \beta_j$ are fitted according to the procedure in (Bollerslev, 1986).

2. The relationship between proxies

The fact that several proxies have been defined for the same latent variable raises the issues of their statistical association. For this reason we computed the proxies, discussed above, on the 40 time series of the French stock market index CAC40 in the period ranging from 05-01-2009 to 22-10-2014 (approximately 6 years). This corresponds to 1489 OHLC (Opening, High, Low, Closing) samples for each time series. Moreover, we obtained the continuously compounded return and the volume variable (representing the number of trades in given trading day).

Figure 1 shows the aggregated correlation (over all the 40 time series) between the proxies, obtained by meta-analysis (Field, 2001). The black rectangles indicate the results of an hierarchical clustering using

(Ward Jr, 1963) with k=3. As expected, we can observe a correlation clustering phenomenon between proxies belonging to the same family, i.e. σ_t^i and $\sigma_t^{SD,n}$. The presence of σ_t^0 in the $\sigma_t^{SD,n}$ cluster can be explained by the fact that the former represents a degenerate case of the latter when $n = 1$. Moreover, we find a correlation between the volume and the σ_t^i family.

3. NARX proxy forecasting

We focus here on the multi-step-ahead forecasting of the proxy σ^G by addressing the question whether a NARX approach can be beneficial in terms of accuracy. In particular we compare a univariate multi-step-ahead NAR model $\sigma_{t+h}^G = f(\sigma_t^G, \cdots, \sigma_{t-m}^G) + \omega$ with a multi-step-ahead NARX model $\sigma_{t+h}^G = f(\sigma_t^G, \cdots, \sigma_{t-m}^G, \sigma_t^X, \cdots, \sigma_{t-m}^X) + \omega$, for a specific embedding order $m = 5$ and for different estimators of the dependency f.

In particular we compare a naive model (average of the past values), a GARCH(1,1), and two machine learning approaches : a feedforward Artificial Neural Networks (single hidden layer, implemented with R `nnet`) and a k-Nearest Neighbors (automatic leave-one-out selection of the number of neighbors). Multi-step-ahead prediction is returned by a direct forecasting strategy (Taieb, 2014). The MASE results (Hyndman and Koehler (2006)) from 10 out-of-sample evaluations (Tashman (2000)) in Table 1 show that both machine learning methods outperform the benchmark methods (naive and GARCH) and that the ANN model can take advantage of the additional information provided by the exogenous proxy. The results in Table 2 confirm that such conclusion remains consistent when moving from a single stock time series in a given market to an index time series (S&P500).

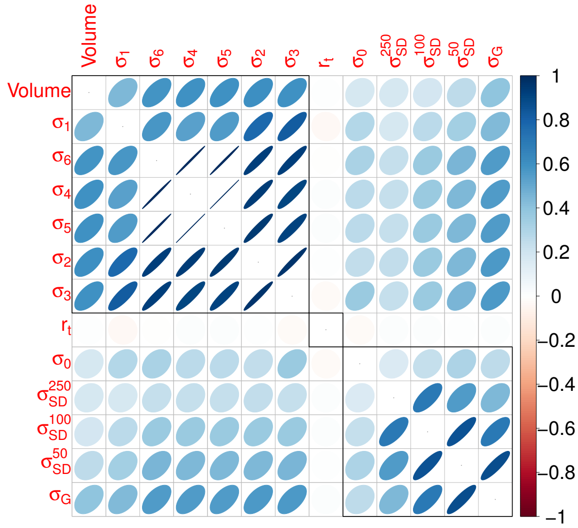

Figure 1. Summary of the correlations between different volatility proxies for the 40 CAC40 time series. Note that the continuously compounded return r_t has a very low correlation with all the other variables.

4. Conclusion and Future work

We studied the relationships between different proxies and we investigated the impact on the accuracy of volatility forecasting of three parameters : the choice of the exogenous proxy, the machine learning technique and the kind of autoregression. Results are preliminary for the moment. For the final version we expect to provide additional comparisons in terms of the number of series, forecasting horizons h model orders m.

Table 1. MASE (normalized wrt the accuracy of a naive method) for a 10-step volatility forecasting horizon on a single stock composing the CAC40 index on the period from 05-01-2009 to 22-10-2014, for different proxy combinations (rows) and different forecasting techniques (columns). The subscript X stands for the NARX model where σ^X is exogenous.

σ^X	ANN	kNN	ANN_X	kNN_X	GARCH(1,1)
σ^6	0.07	0.08	0.06	0.11	1.34
$Volume$	0.07	0.08	0.07	0.14	1.34
$\sigma^{SD,5}$	0.07	0.08	0.07	0.09	1.34
$\sigma^{SD,15}$	0.07	0.08	0.06	0.10	1.34
$\sigma^{SD,21}$	0.07	0.08	0.06	0.10	1.34

Table 2. MASE (normalized wrt the accuracy of a naive method) for a 10-step volatility forecasting horizon on the S&P500 index on the period from 01-04-2012 to 30-07-2013 as in the work of Dash & Dash, 2016, for different proxy combinations (rows) and different forecasting techniques (columns). The subscript X stands for the NARX model where σ^X is exogenous.

σ^X	ANN	kNN	ANN_X	kNN_X	GARCH(1,1)
σ^6	0.58	0.49	0.53	0.56	1.15
$Volume$	0.58	0.49	0.57	0.66	1.15
$\sigma^{SD,5}$	0.58	0.49	0.58	0.58	1.15
$\sigma^{SD,15}$	0.58	0.49	0.65	0.65	1.15
$\sigma^{SD,21}$	0.58	0.49	0.56	0.65	1.15

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